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A Design Method of Multi-mode Multi-Band Bandpass Filters

Pengyu Ma, *Student Member, IEEE,* Bin Wei, *Member, IEEE,* Jiasheng Hong, *Fellow, IEEE,* Zhan Xu, Xubo Guo, *Member, IEEE,* Bisong Cao, and Linan Jiang

***Abstract*—In this paper, a general design method for multi-band filters is proposed. Firstly, the frequency and element transformation from a lowpass prototype filter to a practical multi-band bandpass filter is derived. Afterwards, formulas for extracting the coupling coefficient k between coupled multi-mode resonators and the external quality factor Q_e are also obtained. Thereafter, the design procedure of coupled multi-resonator multi-band filters is similar to that of single-band filters. A tri-band microstrip filter with tri-mode resonators is successfully designed with the proposed method and fabricated, which validates this theory.**

***Index Terms*—Chebyshev filters, coupling matrix for filter synthesis, frequency transformation, multi-bandpass filters, multimode resonators.**

I. Introduction

In MODERN wireless communication, one single transceiver has to operate at multiple frequency bands simultaneously. As an essential part of such systems, multi-band filters are highly needed, due to their miniaturization and high-performance. The design methods had been extensively studied in recent years.

A multi-band filter can be realized by combining different kinds of filters with common ports. In [1], [2], multi-band filters are implemented by paralleling single-band circuits. In [3], a dual-band and a single-band are paralleled to design a tri-band filter. In [4]–[6], they cascade a wide bandpass and some bandstop filters to realize multi-band filters. This method is straightforward but leads to a large circuit size.

Multi-band synthesis technique is also studied recently. With cross-coupling, transmission zeros are introduced to split a single-band into multi-band [7]–[10]. The frequency transformation theory [11] is developed for the multi-band cases [12]–[16]. This method needs special topology. And it is hard to implement the synthesized topology with planar microstrip structure.

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P. Ma, B. Wei, X. Guo, B. Cao and L. Jiang are with the State Key Laboratory of Low-Dimensional Quantum Physics, Department of Physics, Tsinghua University, Beijing 100084, China (e-mail: mapy15@mails.tsinghua.edu.cn; weibin@mail.tsinghua.edu.cn)

J. Hong is with the Institute of Signal, Sensors, and Systems (ISSS), School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh EH14 4AS, U.K. (e-mail: j.hong@hw.ac.uk).

Z. Xu is with the Beijing Key Laboratory of Millimeter Wave and Terahertz Technology, Department of Electronic Engineering, School of Information and Electronics, Beijing Institute of Technology, Beijing 100081, China.

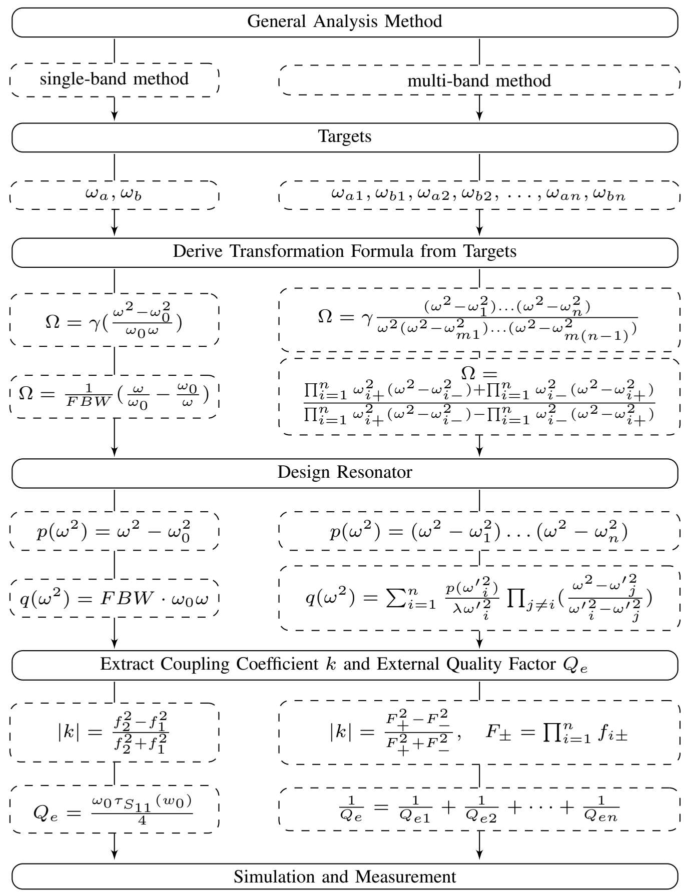


Fig. 1. Comparison between methods for single-band and multi-band in detail.

Using multimode resonators is another popular method. Conventional multi-mode stepped-impedance resonators (SIRs) [17]–[21] and stub-load resonators [22]–[25] are used in multi-band filters. New multi-mode resonators are also proposed recently, such as split-ring resonators [26], [27] and stacked spiral resonators [28]. The circuit size under this method is quite small. But there is a lack of a general design method for coupled multi-mode resonators.

In [29], we already proposed the method to cascade dual-mode resonators. In this paper, we develop the dual-mode method into multi-mode cases, which is useful as an alternative method for designing multi-band filters and based on the theory of coupling matrix. As shown in Fig. 1, the design process of the proposed method is similar to the general design process of single-mode single-band filters. The frequency and element transformations of the n-mode resonator is simplified in this paper under narrowband condition. The coupling coefficient k and external quality factor Q_e between two n-mode resonators are derived using rigorous mathematical tools, such as Vieta's formulas [30], Lagrange polynomial [31] and monic polynomial [32]. A triple-band filter using a triple-mode resonator is designed to validate the proposed method.

II. Frequency Transformation for Multimode Resonators

The frequency and element transformation from a lowpass prototype filter to a single-band bandpass filter is [33, p.53]

$$\Omega = \frac{1}{FBW}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \tag{1}$$

where FBW is the fractional bandwidth and ω_0 is the center angular frequency.

In this section, the frequency and element transformation from a lowpass prototype filter to a multi-band bandpass filter is derived.

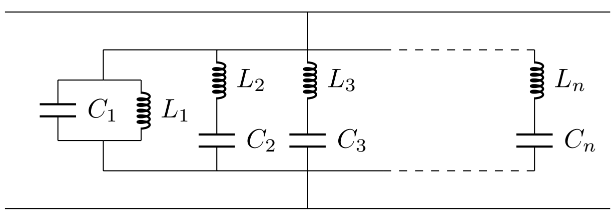


Fig. 2. Topology of multimode resonators

A. Frequency Transformation

Fig. 2 shows the topology of n-mode resonator. Given the narrow range of frequencies, the frequency transformation for n-mode resonators can be simplified as:

$$\begin{aligned}\Omega &= \left(C_1\omega - \frac{1}{L_1\omega} - \sum_{i=2}^{n}\frac{1}{C_i\omega - \frac{1}{L_i\omega}}\right)/Y_0 \\ &= \frac{\omega}{\overline{\omega}}\cdot\frac{\overline{\omega}}{Y_0\omega^2}\left(\frac{C_1L_1\omega^2-1}{L_1} - \sum_{i=2}^{n}\frac{L_i\omega^2}{C_iL_i\omega^2-1}\right) \\ &= \gamma\frac{(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)\ldots(\omega^2-\omega_n^2)}{\omega^2(\omega^2-\omega_{m1}^2)\ldots(\omega^2-\omega_{m(n-1)}^2)}\end{aligned} \tag{2}$$

where Y_0 is the source admittance; γ, $\omega_1, \omega_2, \ldots, \omega_n$, $\omega_{m1}, \ldots, \omega_{m(n-1)}$ are the parameters that define the transformation; $\overline{\omega}$ is the approximate mean value of frequencies. And $\omega/\overline{\omega} \approx 1$ for narrow-range approximation.

Assume

$$p(\omega^2) = (\omega^2-\omega_1^2)(\omega^2-\omega_2^2)\ldots(\omega^2-\omega_n^2) \tag{3}$$

which is a monic polynomial of degree n, and

$$q(\omega^2) = (\omega^2-\omega_{m1}^2)\ldots(\omega^2-\omega_{m(n-1)}^2)/\gamma \tag{4}$$

which is a polynomial of degree $n-1$.

So the Ω can be written as:

$$\Omega = \frac{p(\omega^2)}{\omega^2 q(\omega^2)} \tag{5}$$

For single-mode resonators, (2) can be simplified as:

$$\Omega = \gamma\frac{\omega^2-\omega_1^2}{\omega^2} = \gamma\frac{\omega_0}{\omega}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right) = \frac{1}{FBW}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right) \tag{6}$$

where $\omega_0 = \omega_1$, and narrow-range approximation is used.

For dual-mode resonators, (2) can be simplified as [29]:

$$\Omega = \gamma\frac{(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)}{\omega^2(\omega^2-\omega_{m1}^2)} = \gamma\frac{(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)}{\omega^2(\omega^2-\omega_m^2)} \tag{7}$$

where $\omega_m = \omega_{m1}$.

B. Derivation Frequency Transformation from Target Parameters

The n-passband filters have $2n$ target parameters:

$$\omega_{a1}, \omega_{b1}, \omega_{a2}, \omega_{b2}, \ldots, \omega_{an}, \omega_{bn} \tag{8}$$

where ω_{ai} are the start-frequencies of these n bands and ω_{bi} are the stop-frequencies.

The low-pass prototype response Ω has the following properties at the start-stop frequencies:

$$\begin{cases}|\Omega(\omega_{ai})| = |\Omega(\omega_{bi})| = 1\\ \Omega(\omega_{ai})\Omega(\omega_{bi}) = -1\end{cases} \tag{9}$$

for $i = 1, 2, \ldots, n$.

So there are two situations at each band:

$$\Omega(\omega_{ai}) = 1, \quad \Omega(\omega_{bi}) = -1 \tag{10}$$

or

$$\Omega(\omega_{ai}) = -1, \quad \Omega(\omega_{bi}) = 1 \tag{11}$$

A designer can choose one of these two situations to meet their actual demand. As a general method, we assume:

$$\{\omega_{i+}, \omega_{i-}\} \equiv \{\omega_{ai}, \omega_{bi}\} \tag{12}$$

where ω_{i+} is the frequency that satisfies $\Omega(\omega_{i+}) = 1$, and ω_{i-} is the frequency that satisfies $\Omega(\omega_{i-}) = -1$.

Then,

$$\frac{p(\omega_{i+}^2)}{\omega_{i+}^2 q(\omega_{i+}^2)} = \Omega(\omega_{i+}) = 1 \tag{13}$$

$$\frac{p(\omega_{i-}^2)}{\omega_{i-}^2 q(\omega_{i-}^2)} = \Omega(\omega_{i-}) = -1 \tag{14}$$

From (13),

$$p(\omega_{i+}^2) - \omega_{i+}^2 q(\omega_{i+}^2) = 0 \tag{15}$$

From (14),

$$p(\omega_{i-}^2) + \omega_{i-}^2 q(\omega_{i-}^2) = 0 \tag{16}$$

We assume:

$$\begin{cases}f_1(\omega^2) = p(\omega^2) - \omega^2 q(\omega^2)\\ f_2(\omega^2) = p(\omega^2) + \omega^2 q(\omega^2)\end{cases} \tag{17}$$

From (13) and (14), $\omega_{1+}^2, \omega_{2+}^2, \ldots, \omega_{n+}^2$ are the roots of f_1, and $\omega_{1-}^2, \omega_{2-}^2, \ldots, \omega_{n-}^2$ are the roots of f_2. Therefore, the f_1 and f_2 have these forms:

$$\begin{cases} f_1(\omega^2) = \alpha(\omega^2 - \omega_{1+}^2)(\omega^2 - \omega_{2+}^2)\ldots(\omega^2 - \omega_{n+}^2) \\ f_2(\omega^2) = \beta(\omega^2 - \omega_{1-}^2)(\omega^2 - \omega_{2-}^2)\ldots(\omega^2 - \omega_{n-}^2) \end{cases} \tag{18}$$

Solving (17),

$$\begin{cases} p(\omega^2) = \frac{f_1(\omega^2)+f_2(\omega^2)}{2} \\ q(\omega^2) = \frac{f_2(\omega^2)-f_1(\omega^2)}{2\omega^2} \end{cases} \tag{19}$$

Given that $p(\omega^2)$ is a monic polynomial of degree n and $q(\omega^2)$ is a polynomial of degree $n-1$, we get:

$$\begin{cases} \frac{\alpha+\beta}{2} = 1 \\ \alpha\prod_{i=1}^n \omega_{i+}^2 = \beta\prod_{i=1}^n \omega_{i-}^2 \end{cases} \tag{20}$$

Solving (20)

$$\begin{cases} \alpha = \frac{2\prod_{i=1}^n \omega_{i-}^2}{\prod_{i=1}^n \omega_{i+}^2+\prod_{i=1}^n \omega_{i-}^2} \\ \beta = \frac{2\prod_{i=1}^n \omega_{i+}^2}{\prod_{i=1}^n \omega_{i+}^2+\prod_{i=1}^n \omega_{i-}^2} \end{cases} \tag{21}$$

From (18), (19) and (21),

$$\begin{cases} p(\omega^2) = \frac{\prod_{i=1}^n \omega_{i+}^2(\omega^2-\omega_{i-}^2)+\prod_{i=1}^n \omega_{i-}^2(\omega^2-\omega_{i+}^2)}{\prod_{i=1}^n \omega_{i+}^2+\prod_{i=1}^n \omega_{i-}^2} \\ q(\omega^2) = \frac{\prod_{i=1}^n \omega_{i+}^2(\omega^2-\omega_{i-}^2)-\prod_{i=1}^n \omega_{i-}^2(\omega^2-\omega_{i+}^2)}{\omega^2(\prod_{i=1}^n \omega_{i+}^2+\prod_{i=1}^n \omega_{i-}^2)} \end{cases} \tag{22}$$

And

$$\Omega = \frac{\prod_{i=1}^n \omega_{i+}^2(\omega^2-\omega_{i-}^2)+\prod_{i=1}^n \omega_{i-}^2(\omega^2-\omega_{i+}^2)}{\prod_{i=1}^n \omega_{i+}^2(\omega^2-\omega_{i-}^2)-\prod_{i=1}^n \omega_{i-}^2(\omega^2-\omega_{i+}^2)} \tag{23}$$

III. Formulation for Extracting Parameters

A. Extracting Frequency Transformation

For an n-mode resonator, we all know that $\omega_1, \omega_2, \ldots, \omega_n$ in (2) are the n resonance frequencies. This section proposes a method to extract $\omega_{m1}, \omega_{m2}, \ldots, \omega_{m(n-1)}$ for an n-mode resonator.

From [29], the response of one n-mode resonator has n reflection zeros ($\omega_1, \omega_2, \ldots, \omega_n$) and n transmission zeros ($\omega'_1, \omega', \ldots, \omega'_n$), which satisfy:

$$\Omega({\omega'}_1^2) = \Omega({\omega'}_2^2) = \cdots = \Omega({\omega'}_n^2) \tag{24}$$

Assume:

$$\Omega({\omega'}_i^2) = \frac{p({\omega'}_i^2)}{{\omega'}_i^2 q({\omega'}_i^2)} = \lambda \tag{25}$$

for $i = 1, 2, \ldots, n$.

Therefore,

$$q({\omega'}_i^2) = \frac{p({\omega'}_i^2)}{\lambda{\omega'}_i^2} \tag{26}$$

Given that $q(\omega^2)$ has an $n-1$ degree, we use the Lagrange polynomial:

$$q(\omega^2) = \sum_{i=1}^n \frac{p({\omega'}_i^2)}{\lambda{\omega'}_i^2}\prod_{j\neq i}\left(\frac{\omega^2-{\omega'}_j^2}{{\omega'}_i^2-{\omega'}_j^2}\right) \tag{27}$$

From appendix B, we obtain:

$$\lambda = \gamma\left(1 - \frac{\prod_{i=1}^n \omega_i^2}{\prod_{i=1}^n {\omega'}_i^2}\right) \tag{28}$$

So,

$$\Omega = \gamma\frac{(\prod_{i=1}^n {\omega'}_i^2 - \prod_{i=1}^n \omega_i^2)p(\omega^2)}{\omega^2(\prod_{i=1}^n {\omega'}_i^2)\sum_{i=1}^n \frac{p({\omega'}_i^2)}{{\omega'}_i^2}\prod_{j\neq i}\left(\frac{\omega^2-{\omega'}_j^2}{{\omega'}_i^2-{\omega'}_j^2}\right)} \tag{29}$$

where $\omega_1, \omega_2, \ldots, \omega_n$ are the reflection zeros of the response; $\omega'_1, \omega'_2, \ldots, \omega'_n$ are the transmission zeros; and $p(\omega^2) = (\omega^2-\omega_1^2)(\omega^2-\omega_2^2)\ldots(\omega^2-\omega_n^2)$.

B. Extracting Coupling Coefficient k

The response of coupled two n-mode resonators splits the n peaks into $2n$. From [29], the resultant condition in these peaks satisfies:

$$det\begin{vmatrix}\Omega & m \\ m & \Omega\end{vmatrix} = 0 \tag{30}$$

where m is the normalized coupling coefficient [33, p.196].

Following the definition of coupling coefficient k for single-mode resonators [33, p.196] and dual-mode resonators [29], the multi-mode coupling coefficient k can be defined as:

$$k = \frac{m}{\gamma} \tag{31}$$

From (30) and (31),

$$(\Omega/\gamma)^2 = (m/\gamma)^2 \Longrightarrow \Omega/\gamma = \pm|k| \tag{32}$$

From (2) and (32),

$$(1-|k|)(\omega^2)^n + a_{n-1}(\omega^2)^{n-1} + \cdots + (-1)^n\omega_1^2\omega_2^2\ldots\omega_n^2 = 0 \tag{33}$$

$$(1+|k|)(\omega^2)^n + b_{n-1}(\omega^2)^{n-1} + \cdots + (-1)^n\omega_1^2\omega_2^2\ldots\omega_n^2 = 0 \tag{34}$$

where a_i, b_i are the coefficient.

So (33) and (34) divide the frequencies of the $2n$ peaks into two categories. We assume that $2\pi f_{1+}, 2\pi f_{2+}, \ldots, 2\pi f_{n+}$ represent the positive roots of (33) and $2\pi f_{1-}, 2\pi f_{2-}, \ldots, 2\pi f_{n-}$ represent the positive roots of (34).

Using Vieta's formulas:

$$(2\pi f_{1+})^2(2\pi f_{2+})^2\ldots(2\pi f_{n+}^2) = \frac{\omega_1^2\omega_2^2\ldots\omega_n^2}{1-|k|} \tag{35}$$

$$(2\pi f_{1-})^2(2\pi f_{2-})^2\ldots(2\pi f_{n-})^2 = \frac{\omega_1^2\omega_2^2\ldots\omega_n^2}{1+|k|} \tag{36}$$

(35) then divides (36) into:

$$\frac{f_{1+}^2 f_{2+}^2\cdots f_{n+}^2}{f_{1-}^2 f_{2-}^2\cdots f_{n-}^2} = \frac{1+|k|}{1-|k|} \tag{37}$$

By solving (37), the formula for coupling coefficient k is derived:

$$|k| = \frac{F_+^2 - F_-^2}{F_+^2 + F_-^2} \tag{38}$$

where

$$\begin{cases} F_+ = f_{1+}f_{2+}\cdots f_{n+} \\ F_- = f_{1-}f_{2-}\cdots f_{n-} \end{cases} \tag{39}$$

For single-mode resonators, $F_- = f_1$ and $F_+ = f_2$. (38) can be simplified as:

$$|k| = \frac{f_2^2 - f_1^2}{f_2^2 + f_1^2} \tag{40}$$

which is the conventional formula for coupled single-mode resonators.

For coupled dual-mode resonators, (38) can be simplified as:

$$|k| = \frac{f_{1+}^2 f_{2+}^2 - f_{1-}^2 f_{2-}^2}{f_{1+}^2 f_{2+}^2 + f_{1-}^2 f_{2-}^2} \tag{41}$$

which is the same with the formula in [29].

C. Extracting External Quality Factor Q_e

Using the one-pole matrix [A], the S_{11} is obtained as [29]:

$$S_{11} = \frac{(1-m_{SL}^2)\Omega + 2m_{S1}m_{L1}m_{SL} - \mathrm{j}m_{L1}^2 + \mathrm{j}m_{S1}^2}{-(1+m_{SL}^2)\Omega + 2m_{S1}m_{L1}m_{SL} + \mathrm{j}m_{L1}^2 + \mathrm{j}m_{S1}^2} \tag{42}$$

where m_{SL} is the normalized coupling coefficient between source and load; m_{S1} is the normalized coupling coefficient between source and the resonator; m_{L1} is the normalized coupling coefficient between load and the resonator.

Considering only the coupling between the source and resonator so that $m_{SL} \to 0$ and $m_{L1} \to 0$, we obtain:

$$S_{11} = \frac{\mathrm{j}m_{S1}^2 + \Omega}{\mathrm{j}m_{S1}^2 - \Omega} \tag{43}$$

The group delay of S_{11} can be derived as:

$$\tau_{S_{11}}(\omega) = -\frac{\partial \mathrm{Arg}(S_{11})}{\partial \omega} = 2\cos\left(\frac{\Omega}{m_{S1}^2}\right)\frac{1}{m_{S1}^2}\frac{\partial \Omega}{\partial \omega} \tag{44}$$

Comparing with the formula of Q_e for single mode equation, [33, p.218,229], assume

$$\begin{cases} Q_{ei} = \frac{\omega_i \tau_{S_{11}}(\omega_i)}{4} \quad fori = 1,2,\ldots,n \\ Q_e = \frac{\gamma}{m_{S1}^2} \end{cases} \tag{45}$$

From appendix C,

$$\frac{1}{Q_e} = \frac{1}{Q_{e1}} + \frac{1}{Q_{e2}} + \cdots + \frac{1}{Q_{en}} \tag{46}$$

IV. Numerical Example of A Triband Filter

A. Targets and Transformation

From (2), the transformation for a triband filter can be expressed as:

$$\Omega = \gamma\frac{(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)(\omega^2-\omega_3^2)}{\omega^2(\omega^2-\omega_{m1}^2)(\omega^2-\omega_{m2}^2)} \tag{47}$$

Consider these targets for this triband filter:

$$\begin{aligned} &\omega_{a1} = 2\pi\cdot 4.210 \text{ GHz}, \quad \omega_{b1} = 2\pi\cdot 4.340 \text{ GHz} \\ &\omega_{a2} = 2\pi\cdot 4.595 \text{ GHz}, \quad \omega_{b2} = 2\pi\cdot 4.620 \text{ GHz} \\ &\omega_{a3} = 2\pi\cdot 4.895 \text{ GHz}, \quad \omega_{b3} = 2\pi\cdot 4.920 \text{ GHz} \end{aligned} \tag{48}$$

TABLE I
Four-resonator Chebyshev Coupling Matrix with 20-dB in-band Return Loss

0	1.0352	0	0	0	0
1.0352	0	0.9106	0	0	0
0	0.9106	0	0.6999	0	0
0	0	0.6999	0	0.9106	0
0	0	0	0.9106	0	1.0352
0	0	0	0	1.0352	0

To enhance the isolation of this filter higher, one way is using ω_{m1} and ω_{m2} isolate these passbands. Therefore, we ask:

$$\omega_1 < \omega_{m1} < \omega_2 < \omega_{m2} < \omega_3 \tag{49}$$

Under this condition, the $\omega_{i\pm}$ defined in (12) can be expressed as:

$$\begin{aligned} &\omega_{1-} = \omega_{a1}, \quad \omega_{1+} = \omega_{b1}, \quad \omega_{2-} = \omega_{a2} \\ &\omega_{2+} = \omega_{b2}, \quad \omega_{3-} = \omega_{a3}, \quad \omega_{3+} = \omega_{b3} \end{aligned} \tag{50}$$

From (23),

$$\begin{aligned} \Omega &= \frac{\prod_{i=1}^3 \omega_{i+}^2(\omega^2-\omega_{i-}^2) + \prod_{i=1}^3 \omega_{i-}^2(\omega^2-\omega_{i+}^2)}{\prod_{i=1}^3 \omega_{i+}^2(\omega^2-\omega_{i-}^2) - \prod_{i=1}^3 \omega_{i-}^2(\omega^2-\omega_{i+}^2)} \\ &= \frac{\prod_{i=1}^3 \omega_{bi}^2(\omega^2-\omega_{ai}^2) + \prod_{i=1}^3 \omega_{ai}^2(\omega^2-\omega_{bi}^2)}{\prod_{i=1}^3 \omega_{bi}^2(\omega^2-\omega_{ai}^2) - \prod_{i=1}^3 \omega_{ai}^2(\omega^2-\omega_{bi}^2)} \\ &= 24.4\frac{\omega^6 - 64\cdot(2\pi)^2\omega^4 + 1338\cdot(2\pi)^4\omega^2 - 9334\cdot(2\pi)^6}{\omega^6 - 44\cdot(2\pi)^2\omega^4 + 486\cdot(2\pi)^4\omega^2} \\ &= \gamma\frac{(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)(\omega^2-\omega_3^2)}{\omega^2(\omega^2-\omega_{m1}^2)(\omega^2-\omega_{m2}^2)} \end{aligned} \tag{51}$$

where

$$\begin{aligned} &\omega_1 = 2\pi\cdot 4.277 \text{ GHz}, \quad \omega_2 = 2\pi\cdot 4.605 \text{ GHz} \\ &\omega_3 = 2\pi\cdot 4.905 \text{ GHz}, \quad \omega_{m1} = 2\pi\cdot 4.553 \text{ GHz} \\ &\omega_{m2} = 2\pi\cdot 4.844 \text{ GHz}, \quad \gamma = 24.4 \end{aligned} \tag{52}$$

The lossless coupling matrix of four-resonator Chebyshev prototype response with 20-dB in-band return loss is extracted using the technique in [34] and shown in TABLE I.

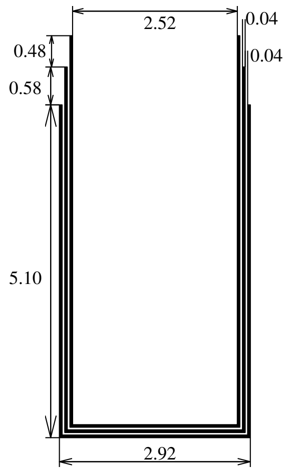
From (31) and (45), the k and Q_e are derived: $k_{12} = k_{34} = 0.0372$, $k_{23} = 0.0286$, $Q_e = 22.76$

B. Resonator Designation

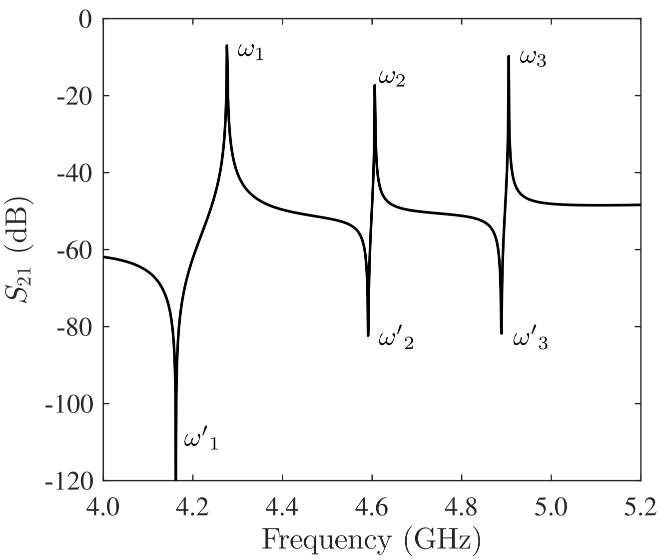
$\omega_1, \omega_2, \omega_3, \omega_{m1}, \omega_{m2}$ determine the property of the triple-mode resonator. Three closed coupled hairpin resonators can form a tri-mode resonator. It seems that arm lengths and bottom lengths mainly affect the ω_1, ω_2, ω_3, while the distances between these hairpin resonators mainly affect the ω_{m1}, ω_{m2}. We carefully optimize all these parameters to meet our requirements. Fig. 3(a) shows the structure that satisfies these conditions, and Fig. 3(b) shows the response of this resonator, where we get that: $\omega_1 = 2\pi\cdot 4.276$ GHz, $\omega_2 = 2\pi\cdot 4.606$ GHz, $\omega_3 = 2\pi\cdot 4.905$ GHz, $\omega'_1 = 2\pi\cdot 4.162$ GHz, $\omega'_2 = 2\pi\cdot 4.591$ GHz, $\omega'_3 = 2\pi\cdot 4.889$ GHz.

ω_{m1} and ω_{m2} are derived using (29) as:

$$\omega_{m1} = 2\pi\cdot 4.843 \text{ GHz}, \quad \omega_{m2} = 2\pi\cdot 4.555 \text{ GHz} \tag{53}$$



(a)



(b)

Fig. 3. (a) Layout of the triple-mode resonator (dimensions in mm). (b) S_{21}-response of single resonator, where $\omega_1 = 2\pi \cdot 4.276$ GHz, $\omega_2 = 2\pi \cdot 4.606$ GHz, $\omega_3 = 2\pi \cdot 4.905$ GHz, $\omega'_1 = 2\pi \cdot 4.162$ GHz, $\omega'_2 = 2\pi \cdot 4.591$ GHz, $\omega'_3 = 2\pi \cdot 4.889$ GHz

They are close to the parameters in (52) and within the permissible range.

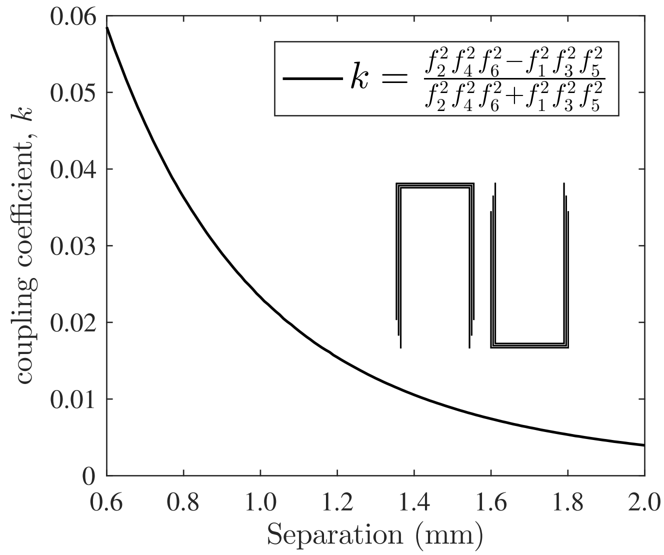
C. Simulation and Analysis

1) Extracting k: From (38), the formula for coupling coefficient of triple-mode resonators is expressed as:

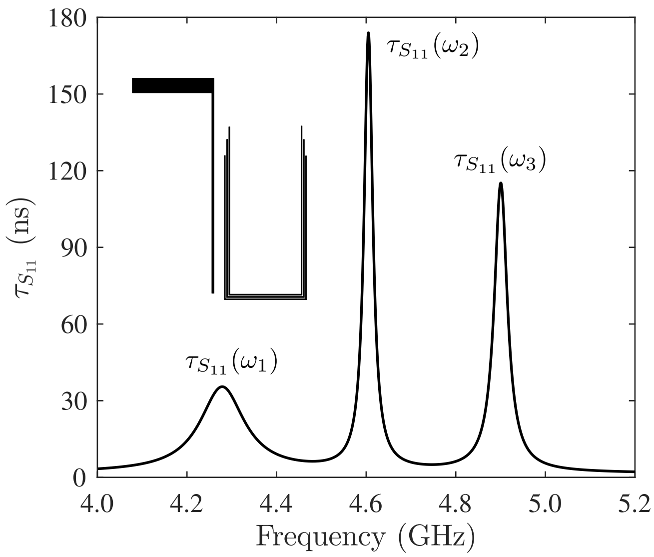
$$|k| = \frac{f_{1+}^2 f_{2+}^2 f_{3+}^2 - f_{1-}^2 f_{2-}^2 f_{3-}^2}{f_{1+}^2 f_{2+}^2 f_{3+}^2 + f_{1-}^2 f_{2-}^2 f_{3-}^2} \tag{54}$$

When two triple-mode resonators are coupled together, six peaks appear and are marked as f_1, f_2, f_3, f_4, f_5, f_6 arranged from small to large. Following (49), we know:

$$\begin{aligned} f_{1+} &= f_2, \quad f_{2+} = f_4, \quad f_{3+} = f_6 \\ f_{1-} &= f_1, \quad f_{2-} = f_3, \quad f_{3-} = f_5 \end{aligned} \tag{55}$$



(a)



(b)

Fig. 4. (a) Variation of coupling coefficient with separation. (b) Variation of group delay with frequency, where $\tau_{S_{11}}(\omega_1) = 35.48$ ns, $\tau_{S_{11}}(\omega_2) = 173.99$ns, $\tau_{S_{11}}(\omega_3) = 115.18$ ns.

So the coupling coefficient k is derived:

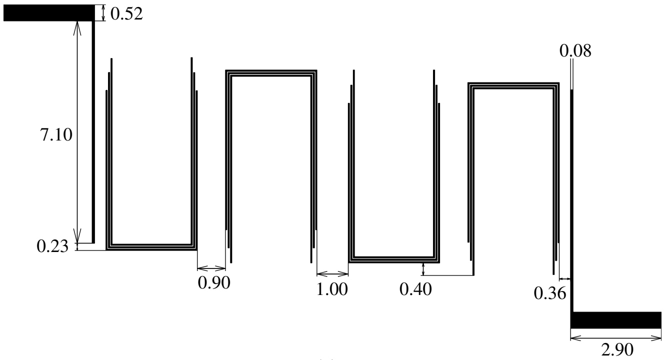
$$|k| = \frac{f_2^2 f_4^2 f_6^2 - f_1^2 f_3^2 f_5^2}{f_2^2 f_4^2 f_6^2 + f_1^2 f_3^2 f_5^2} \tag{56}$$

The variation of coupling coefficient with separation is shown in Fig. 4(a).

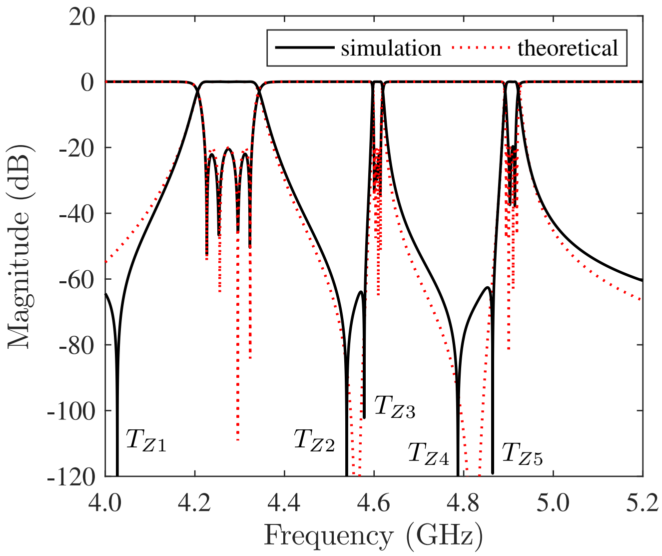
2) Extracting Q_e: Fig. 4(b) shows the I/O coupling structure and the group delay response of S_{11}. The group delay-s at there three resonant frequencies equal 35.48 ns, 173.99 ns, 115.18 ns. From (46), the external quality factor is derived:

$$Q_e = 26.02 \tag{57}$$

which is within the permissible range.

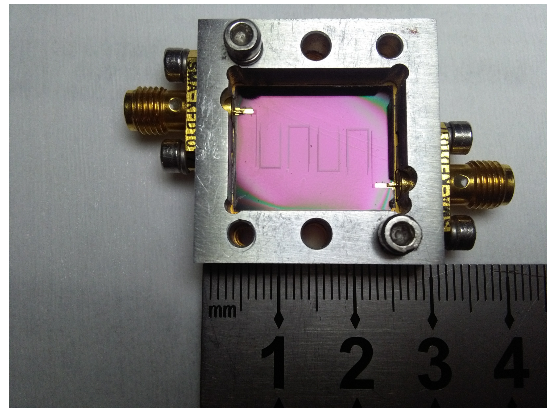


(a)

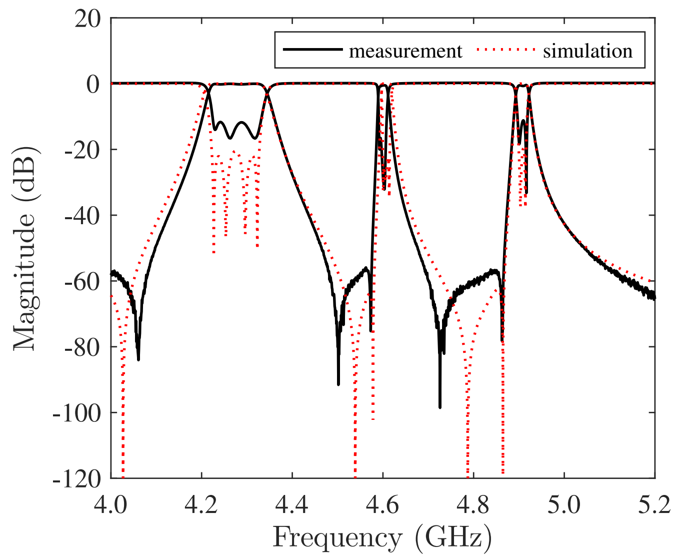


(b)

Fig. 5. (a) Layout of the designed filter. (dimensions in mm) (b) Simulation and theoretical result.



(a)



(b)

Fig. 6. (a) Photograph of the filter. (b) Simulated and measured response.

3) Layout and Analysis: Fig. 5(a) shows the filter design, while Fig. 5(b) shows the simulation, compared with the theoretical response, which is derived by the coupling matrix, directly. Because of the cross-couplings, five transmission zeros: T_{Z1}, T_{Z2}, T_{Z3}, T_{Z4}, T_{Z5} are found in the simulated S21-response. Similar to [29], Ω is the same at T_{Z1}, T_{Z3} and T_{Z5} or T_{Z2} and T_{Z4}. In other words, $\Omega(T_{Z1}) = \Omega(T_{Z3}) = \Omega(T_{Z5})$ and $\Omega(T_{Z2}) = \Omega(T_{Z4})$.

D. Fabrication and Measurement

The filter is fabricated on a 0.51-mm thick MgO substrate with two-side 500nm-thick YBCO HTS films. Photolithography and ion beam etching are performed in the fabrication process. Fig. 6(a) shows the filter, which is measured by an Agilent E5072A network analyzer, while Fig. 6(b) shows the measured and simulated results. Because of the fabrication and measurement process, the measured responses may differ from the simulated. The measured return loss is better than 11dB.

V. Conclusion

This study uses the theory of coupling matrix and rigorous mathematical tools to develop a general method for multi-mode multi-bandpass filters. This method includes the conventional methods for single-bandpass filters and dual-bandpass filters. This paper also presents a simplified transformation formula for multimode resonators and formulas for extracting coupling coefficient k and external quality factor Q_e. A triple-mode resonator is studied and used for the triple-bandpass filter designation, which validates this theory.

Appendix A
Definition of M-function and Its Property

To facilitate the derivations in following appendixes, we define an M-function:

$$M^{l}_{(a_1,a_2,\ldots,a_n)} = \sum_{i=1}^{n} \frac{a_i^l}{\prod_{j\neq i}(a_i - a_j)} \tag{58}$$

where $l \in N, a_i \neq a_j, \forall i \neq j$.

The M-function has this property:

$$M^{l}_{(a_1,a_2,\ldots,a_n)} = \begin{cases} 0 & l < n-1 \\ 1 & l = n-1 \end{cases} \tag{59}$$

The proof of this property is followed:
By considering:

$$f(x) = x^l \tag{60}$$

We obtain,

$$f(a_1) = a_1^l, f(a_2) = a_2^l, \ldots, f(a_n) = a_n^l \tag{61}$$

When $l < n$, we use the Lagrange polynomial:

$$f(x) = \sum_{i=1}^{n} a_i^l \prod_{j\neq i}(\frac{x-a_j}{a_i-a_j}) = x^l \tag{62}$$

By comparing the coefficient of x^{n-1}, we obtain:

$$M^l_{(a_1,a_2,\ldots,a_n)} = \begin{cases} 0 & l < n-1 \\ 1 & l = n-1 \end{cases} \tag{63}$$

APPENDIX B
DRIVE λ IN (27)

This section needs the definition of M-function and it property in appendix A.

Comparing the leading coefficient of $q(\omega^2)$ in (27) and (4),

$$\sum_{i=1}^{n} \frac{p({\omega'}_i^2)}{\lambda {\omega'}_i^2 \prod_{j\neq i}({\omega'}_i^2 - {\omega'}_j^2)} = \frac{1}{\gamma} \tag{64}$$

Then,

$$\begin{aligned} \frac{\lambda}{\gamma} &= \sum_{i=1}^{n} \frac{({\omega'}_i^2-\omega_1^2)({\omega'}_i^2-\omega_2^2)\ldots({\omega'}_i^2-\omega_n^2)}{{\omega'}_i^2 \prod_{j\neq i}({\omega'}_i^2 - {\omega'}_j^2)} \\ &= \sum_{i=1}^{n} \frac{({\omega'}_i^2)^n}{{\omega'}_i^2 \prod_{j\neq i}({\omega'}_i^2 - {\omega'}_j^2)} \\ &\quad - (\sum_{i=1}^{n}\omega_i^2)(\sum_{i=1}^{n} \frac{({\omega'}_i^2)^{n-1}}{{\omega'}_i^2 \prod_{j\neq i}({\omega'}_i^2 - {\omega'}_j^2)}) \\ &\quad + (\sum_{i\neq j}\omega_i^2\omega_j^2)(\sum_{i=1}^{n} \frac{({\omega'}_i^2)^{n-1}}{{\omega'}_i^2 \prod_{j\neq i}({\omega'}_i^2 - {\omega'}_j^2)}) + \ldots \\ &\quad + (-1)^n(\prod_{i=1}^{n}\omega_i^2)(\sum_{i=1}^{n} \frac{1}{{\omega'}_i^2 \prod_{j\neq i}({\omega'}_i^2 - {\omega'}_j^2)}) \\ &= M^n_{(0,{\omega'}_1^2,\ldots,{\omega'}_n^2)} - (\sum_{i=1}^{n}\omega_i^2)M^{n-1}_{(0,{\omega'}_1^2,\ldots,{\omega'}_n^2)} \\ &\quad + (\sum_{i\neq j}\omega_i^2\omega_j^2)M^{n-2}_{(0,{\omega'}_1^2,\ldots,{\omega'}_n^2)} + \ldots \\ &\quad + (-1)^n(\prod_{i=1}^{n}\omega_i^2)(M^0_{(0,{\omega'}_1^2,\ldots,{\omega'}_n^2)} - \frac{(-1)^n}{\prod_{i=1}^{n}{\omega'}_i^2}) \\ &= 1 - \frac{\prod_{i=1}^{n}\omega_i^2}{\prod_{i=1}^{n}{\omega'}_i^2} \end{aligned} \tag{65}$$

APPENDIX C
PROOF OF $\frac{1}{Q_e} = \frac{1}{Q_{e1}} + \frac{1}{Q_{e2}} + \cdots + \frac{1}{Q_{en}}$

This section needs the definition of M-function and it property in appendix A.

Calculate the group-delay in (44) as:

$$\begin{aligned} \tau_{S_{11}}(\omega_i) &= -\frac{\partial \mathrm{Arg}(S_{11})}{\partial \omega}\bigg|_{\omega=\omega_i} \\ &= 2\cos(\frac{\Omega(\omega_i)}{m_{S1}^2})\frac{1}{m_{S1}^2}\frac{\partial \Omega}{\partial \omega}\bigg|_{\omega=\omega_i} \\ &= \frac{4\gamma\prod_{j\neq i}(\omega_i^2-\omega_j^2)}{\omega_i m_{S1}^2 \prod_{j=1}^{n-1}(\omega_i^2-\omega_{mj}^2)} \end{aligned} \tag{66}$$

From (45),

$$Q_{ei} = \frac{\gamma\prod_{j\neq i}(\omega_i^2-\omega_j^2)}{m_{S1}^2\prod_{j=1}^{n-1}(\omega_i^2-\omega_{mj}^2)} = Q_e\frac{\prod_{j=1}^{n-1}(\omega_i^2-\omega_{mj}^2)}{\prod_{j\neq i}(\omega_i^2-\omega_j^2)} \tag{67}$$

Then,

$$\frac{Q_e}{Q_{ei}} = \frac{\prod_{j=1}^{n-1}(\omega_i^2-\omega_{mj}^2)}{\prod_{j\neq i}(\omega_i^2-\omega_j^2)} \tag{68}$$

Then,

$$\begin{aligned} Q_e\sum_{i=1}^{n}\frac{1}{Q_{ei}} &= \sum_{i=1}^{n}\frac{\prod_{j=1}^{n-1}(\omega_i^2-\omega_{mj}^2)}{\prod_{j\neq i}(\omega_i^2-\omega_j^2)} \\ &= \sum_{i=1}^{n}\frac{(\omega_i^2)^{n-1}}{\prod_{j\neq i}(\omega_i^2-\omega_j^2)} \\ &\quad - (\sum_{i=1}^{n-1}\omega_{mi}^2)\sum_{i=1}^{n}\frac{(\omega_i^2)^{n-2}}{\prod_{j\neq i}(\omega_i^2-\omega_j^2)} \\ &\quad + (\sum_{i\neq j}\omega_{mi}^2\omega_{mj}^2)\sum_{i=1}^{n}\frac{(\omega_i^2)^{n-3}}{\prod_{j\neq i}(\omega_i^2-\omega_j^2)} + \ldots \\ &\quad + (-1)^{n-1}(\prod_{i=1}^{n-1}\omega_{mi}^2)\sum_{i=1}^{n}\frac{1}{\prod_{j\neq i}(\omega_i^2-\omega_j^2)} \\ &= M^{n-1}_{(\omega_1^2,\omega_2^2,\ldots,\omega_n^2)} - (\sum_{i=1}^{n-1}\omega_{mi}^2)M^{n-2}_{(\omega_1^2,\omega_2^2,\ldots,\omega_n^2)} \\ &\quad + (\sum_{i\neq j}\omega_{mi}^2\omega_{mj}^2)M^{n-3}_{(\omega_1^2,\omega_2^2,\ldots,\omega_n^2)} + \ldots \\ &\quad + (-1)^{n-1}M^0_{(\omega_1^2,\omega_2^2,\ldots,\omega_n^2)} \\ &= 1 \end{aligned} \tag{69}$$

So,

$$\frac{1}{Q_e} = \frac{1}{Q_{e1}} + \frac{1}{Q_{e2}} + \cdots + \frac{1}{Q_{en}} \tag{70}$$

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Pengyu Ma (S'16) was born in Henan, China, in 1993. He received the B.Sc. degree in applied physics from Tsinghua University, Beijing, China, in 2015. He is currently working toward Ph.D. degree in the Department of Physics, Tsinghua University, Beijing, China.

He is a reviewer for IEEE Transactions on Microwave Theory and Techniques. His current research interests include high-temperature superconducting materials and their application, RF and microwave circuits, theory of coupling matrix.

Bin Wei (M'12) was born in Zhejiang, China, in 1974. He received the B.Sc. and Ph.D. degrees in physics from Tsinghua University, Beijing, China, in 1996 and 2003, respectively. Since 2008, he has been an Associate Professor with the Department of Physics, Tsinghua University.

His current research interests include high-temperature superconducting microwave filters, HTS filter subsystems and their applications in wireless communications systems.

Jiasheng Hong (M'94-SM'05-F'12) received the D.Phil. degree in engineering science from the University of Oxford, Oxford, U.K., in 1994. His doctoral dissertation concerned EM theory and applications. In 1994, he joined the University of Birmingham, Birmingham, U.K., where he was involved with microwave applications of high-temperature superconductors, EM modeling, and circuit optimization. In 2001, he joined the Department of Electrical, Electronic and Computer Engineering, Heriot-Watt University, Edinburgh, U.K., and is currently a Professor leading a team for research into advanced RF/microwave device technologies. He has authored and coauthored over 200 journal and conference papers, and two books, Microstrip Filters for RF/Microwave Applications (Wiley, 1st ed., 2001, 2nd ed., 2011) and RF and Microwave Coupled-Line Circuits (Artech House, 2nd ed., 2007). His current interests involve RF/microwave devices, such as antennas and filters, for wireless communications and radar systems, as well as novel material and device technologies including multilayer circuit technologies using package materials such as liquid crystal polymer, RF MEMS, ferroelectric and high-temperature superconducting devices.

Zhan Xu was born in Hunan, China, in 1987. He received the B.Sc. and Ph.D. degrees in physics from Tsinghua University, Beijing, China, in 2009 and 2014, respectively.

Since 2014, he has been with the Beijing Institute of Technology, Beijing, China. His research interests include high-temperature superconducting devices and applications.

Xubo Guo (M'12) was born in Shanxi, China, in 1981. He received the B.Sc., M.Sc., and Ph.D. degrees in physics from Tsinghua University, Beijing, China, in 2001, 2004, and 2008, respectively. His doctoral research concerned high-power HTS filters.

From July 2008 to June 2010, he was a Postdoctoral Researcher with the Department of Electronic Engineering, Tsinghua University. Since July 2010, he has been an Engineer with the Department of Physics, Tsinghua University. He has authored and co-authored over 50 technical papers. His current research interests include HTS filters from VHF to Ka-band, as well as development and applications of HTS filter subsystems for mobile communications base stations and wireless receivers.

Bisong Cao was born in Jiangsu, China, in 1946. He received the B.Sc. and M.Sc. degrees in engineering physics from Tsinghua University, Beijing, China, in 1970 and 1981, respectively, and the Ph.D. degree in materials science from Tokyo University, Tokyo, Japan, in 1989.

He has been with the Department of Physics, Tsinghua University, since 1981, where he is currently a Professor. His current research interests include HTS physics and HTS microwave devices and their applications. He has published more than 100 papers in journals and international conference proceedings.

Prof. Cao is a Committee Member of the Superconductor Electronics Branch, Chinese Society for Electronics, and of the Low Temperature Physics Branch, Chinese Society for Physics.

Linan Jiang was born in Beijing, China, in 1986. He received the B.Sc. degree in applied physics from Beijing University of Technology, Beijing, China, in 2009.

Since 2009, he has been an Assistant Engineer with the Department of Physics, Tsinghua University, Beijing. His current research interests include high-temperature superconducting (HTS) microwave filters fabrication and HTS filter subsystems design and integration.